A Magnetic Field and Electric Circuit Hybrid Coupled Transient Finite Element Method and its Application to Electric Machines

S. L. Ho[1], Shuangxia Niu[1], H. L. Li[1], W. N. Fu[1], and Jianguo Zhu[2]
[1]Department of Electrical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong
[2] Faculty of Engineering and Information Technology, University of Technology, Sydney, Australia
eesxniu@polyu.edu.hk

Abstract — A hybrid coupling algorithm to model the transient process of magnetic field – electric circuit systems using sub-matrix method is presented. It inherits the advantages of both direct coupling and indirect coupling methods. A special formulation of time stepping finite-element method (FEM) for this algorithm is presented, which allows winding currents are used as unknowns while using nodal method for external circuit equations. The merit is that the structure of the coefficient matrix of the system equations can be kept the same for field – circuit direct coupling and equivalent parameter extraction. The method is applied to simulate power electronic motor drives.

***Index Terms* — Coupling, electric circuit, electric motor, finite-element method, power electronic circuit.**

I. INTRODUCTION

To simulate the dynamic operations of power electronic circuit driven motors using numerical methods, usually the magnetic field, electric circuit, and mechanical movement are coupled together [1-2]. Because of the large inertia, and hence the relatively large time constant in mechanical systems, the precision is not unduly affected if the mechanical system is coupled with a delay of one time step. For magnetic field and electric circuit, the transient process is changing quickly. A one step delay in the coupling may produce very large numerical error. There are basically two methods to couple the magnetic field and electric circuit, namely direct coupling and indirect coupling [3-4]. Direct coupling has high accuracy but it may not converge if the circuit has highly nonlinear power electronic switching elements. Indirect coupling has good convergence but there is a one step delay in the parameters from electric circuit or magnetic field or there is an additional iteration loop.

In this paper a hybrid method for magnetic field and electric circuit coupling using finite-element method (FEM) with sub-matrix technique is proposed. Power electronic driven electric machines are used as examples to show the method. The coupling rules presented in this paper are:

(1) All stranded windings with voltage and current sources, solid conductors with voltage and current sources, star-connection of multi-phase windings, end-connected conductors such as squirrel cages, are connected as internal direct coupling. The formulations are chosen according to principle that the number of unknowns is smallest.

(2) In the external circuit if there are nonlinear elements such as power electronic switching elements, to insure the convergence of the nonlinear iterations at each time step, an indirect coupling method is first used to predict the initial switching status. The magnetic field equations are then coupled with the power electronic circuit equations directly and the complete system equations are solved simultaneously. A special FEM formulation for this algorithm, in that the coefficient matrix of the system equations are kept the same regardless of whether direct or indirect coupling is used, is reported in this paper.

(3) To avoid that the rotor's position is also an unknown during nonlinear iteration and re-mesh at each nonlinear iteration step, the rotor's position is first predicted. After the finish of each time step, its value is adjusted to increase its accuracy.

In the paper a method to extract the equipment circuit parameters of the magnetic field for the indirect coupling is also presented.

II. INDIRECT AND DIRECT COUPLING METHOD

If the circuit scale is large or it has nonlinear elements, the field equations and the circuit equations can be solved separately, and this is called indirect coupling. The indirect coupling method can be classified as source coupling and parameter coupling. In source coupling, when the circuit is solved, the windings and conductors in the coupling ports are replaced by a current source or a voltage source. For source coupling there is only one degree of freedom (DoF) in the winding; either voltage or current is fixed. If, for example, the source is voltage, then only the current will be solved.

Another indirect coupling method is parameter coupling. In parameter coupling, when the circuit is solved, the windings and conductors in the coupling ports are replaced by a series of resistance, inductance and back emf. There are two DoFs in the winding because both voltage and current will be solved. The parameter coupling has a higher precision compared to that obtained from the source coupling method. The drawback is that the field equations and circuit equations are solved separately and hence the field parameter has one step delay.

If the circuit scale is small and it has no nonlinear elements, the field equations and the circuit equations can be solved simultaneously, which is called direct coupling. Here the circuit formulation is based on the nodal method, which is mostly used in circuit simulation programs.

III. HYBRID COUPLING METHOD

In direct coupling method, the parameters of the magnetic field cannot be computed, because currents of the windings and solid conductors in the field regions which are connected with the external circuit are not unknowns. Therefore, for hybrid coupling, it is necessary to deduce a new formulation that uses nodal method for the external circuit whilst keeping the winding currents as unknowns.

The branch voltage $\boldsymbol{u}_b$ consists of $\boldsymbol{u}_w$ and $\boldsymbol{u}_e$, which can be separated into the following equations:

$$\{\boldsymbol{u}_w^k\} = [\boldsymbol{A}_{nw}^T]\{\boldsymbol{u}_n^k\} \quad (1)$$

$$\{\boldsymbol{u}_e^k\} = [\boldsymbol{A}_{ne}^T]\{\boldsymbol{u}_n^k\} \quad (2)$$

where $\boldsymbol{A}_{nw}$ and $\boldsymbol{A}_{ne}$ are the node-to-branch incidence matrixes associated with $\boldsymbol{u}_w$ and $\boldsymbol{u}_e$, respectively. The Kirchhoff's current law becomes,

$$\begin{bmatrix}\boldsymbol{A}_{nw} & \boldsymbol{A}_{ne}\end{bmatrix}\begin{Bmatrix}\boldsymbol{i}_w^k \\ \boldsymbol{i}_e^k\end{Bmatrix} = 0 \quad (3)$$

From the above equation, one has:

$$[\boldsymbol{A}_{ne}]\{\boldsymbol{i}_e^k\} = -[\boldsymbol{A}_{nw}]\{\boldsymbol{i}_w^k\} \quad (4)$$

Substituting these relationships into the system equations, the final global equations are obtained:

$$\begin{bmatrix} \boldsymbol{C}_{11} + \frac{\boldsymbol{D}_{11}}{\Delta t} & -\boldsymbol{C}_{12} & -\boldsymbol{C}_{13} & 0 \\ -\boldsymbol{C}_{12}^T & \boldsymbol{C}_{22} & 0 & 0 \\ -\boldsymbol{C}_{13}^T & 0 & -\Delta t\boldsymbol{C}_{33} & \Delta t\boldsymbol{A}_{nw}^T \\ 0 & 0 & \Delta t\boldsymbol{A}_{nw} & \boldsymbol{A}_{ne}\Delta t\boldsymbol{G}_e\boldsymbol{A}_{ne}^T \end{bmatrix}\begin{Bmatrix}\boldsymbol{A}^k \\ \boldsymbol{i}_{ad}^k \\ \boldsymbol{i}_w^k \\ \boldsymbol{u}_n^k\end{Bmatrix} = \begin{Bmatrix} \boldsymbol{P}_1 + \frac{\boldsymbol{D}_{11}}{\Delta t}\boldsymbol{A}^{k-1} \\ -\boldsymbol{C}_{12}^T\boldsymbol{A}^{k-1} \\ -\boldsymbol{C}_{13}^T\boldsymbol{A}^{k-1} \\ \Delta t\boldsymbol{A}_{ne}\boldsymbol{P}_e \end{Bmatrix} \quad (5)$$

This formulation uses the nodal voltages as unknowns in the circuit and keeps the winding currents as unknowns. Hence the same coefficient matrix in (5) can be used to extract the parameters for indirect coupling. Equation (5) is solved at the FEM solver. At each nonlinear iteration step, the sub-matrixes of the circuit are transferred from the circuit solver to allow the FEM solver and the circuit solver to be developed independently.

In the proposed method, the indirect coupling method is first used to determine the status of the switching elements. Then, using the direct coupling formulation (5), the field equations and circuit equations are solved simultaneously. To insure the convergence of the iteration when solving the nonlinear system equations, the status of the switching elements is frozen. After the nonlinear iteration, the status of the switching elements is checked. If the status is changed, the process has to rollback and the time step will be reduced.

The procedure at each time step t^k is:

(1) According to the field parameters at t^{k-1}, simulate the circuit from t^{k-1} to t^k; determine the status of the switching elements. If the circuit solver detects that the status of the switching elements is changed between t^{k-1} to t^k, the time step size Δt^k is deduced and the new time will be $t^k = t^{k-1} + \Delta t^k$.

(2) Solve the field – circuit coupled equations (5). After the nonlinear iteration, check the status of the switching elements. If the status of the switching elements has changed, reduce the time step size, rollback to step (1).

(3) Extract the magnetic field parameters at t^k.

(4) Go to the next step.

IV. A Test Case

The method has been applied to simulate an inverter driven 24 V three phase-winding brushless d.c. motor, using Nd-Fe-B as excitation (Fig. 1). By comparing between the computed and measured phase currents as shown in Fig. 2, the proposed method is verified.

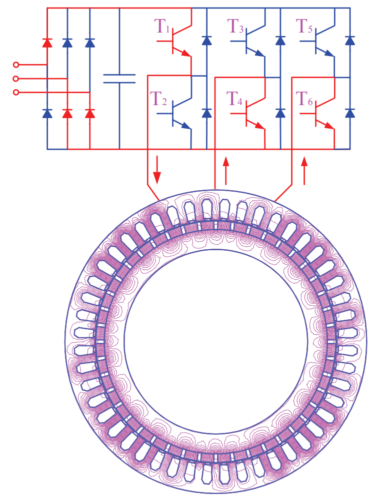

Fig. 1. A brushless d.c. motor drive.

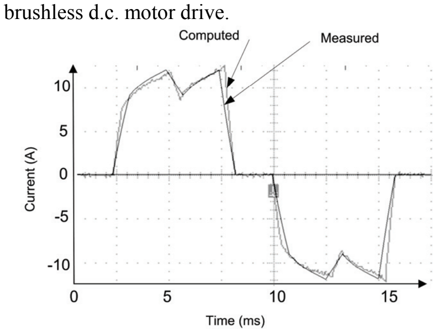

Fig. 2. Computed and measured phase currents.

V. Conclusion

The proposed hybrid method can directly couple the magnetic field and electric circuit together, and also avoid the non-convergence problem in the standard direct coupling method. The proposed formulation for hybrid coupling is not just being used for direct coupling, and it can also be used for parameter extraction of the magnetic field system.

References

[1] N. A. Demerdash, J. F. Bangura, and A. A. Arkadan, "A Time-stepping Coupled Finite Element-state Space Model for Induction Motor Drives—Part 1: Model Formulation and Machine Parameter Computation," *IEEE Trans. Energy Conversion*, Vol. 14, No. 4, Dec. 1999, pp. 1465–1471.

[2] J. Vaananen, "Circuit Theoretical Approach to Couple Two-dimensional Finite Element Models with External Circuit Equations," *IEEE Trans. Magn.*, Vol. 32, No. 2, Mar. 1996, pp. 400–410.

[3] Kay Hameyer, Johan Driesen, Herbert De Gersem and Ronnie Belmans, "The Classification of Coupled Field Problems, " *IEEE Trans. on Magn.*, Vol. 35, No. 3, May 1999, pp.1618-1621.

[4] I.A. Tsukerman, A. Konrad, G. Meunier, J.C. Sabonnadiere, "Coupled Field-circuit Problems: Trends and Accomplishments," *IEEE Trans. Magn.* Vol. 29, No. 2, Mar 1993, pp.1701-1704.